Grade 6 Math Circles

March 4th-8th, 2024

The Binomial Coefficient - Problem Set

Note: Problems that are marked with * are considered challenge problems!

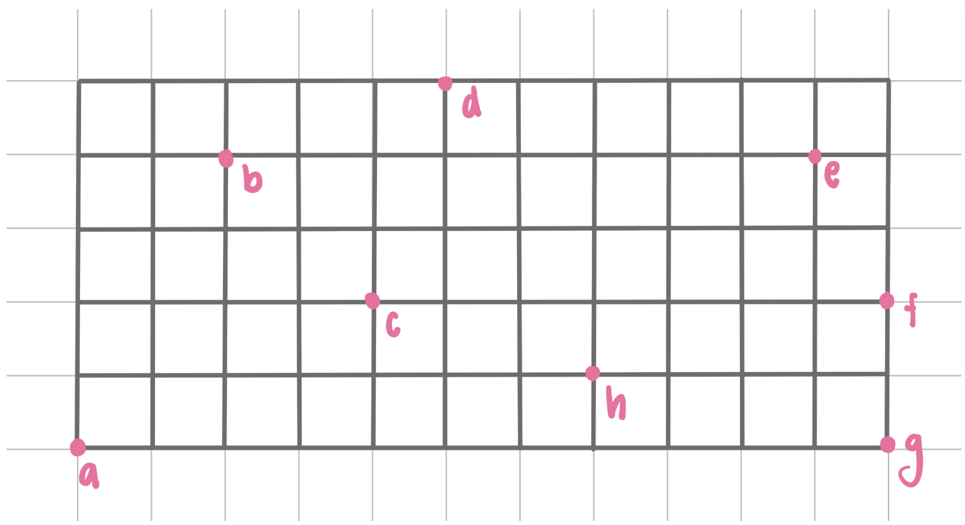
1. Evaluate the following factorials:
   (a) 7!
   (b) 9!
   (c) 10!
2. Reduce the following fractions to lowest terms:
   (a) 155/225
   (b) 20/290
   (c) 252/369
3. Evaluate the following quotients of factorials:
   (a) 5!/2!
   (b) 7!/9!
   (c) 12!/5!
4. Rewrite the following products as quotients of factorials:
   (a) $5 \times 4 \times 3$
   (b) $8 \times 7 \times 6 \times 5$
   (c) $13 \times 12 \times 11$
   (d) 12
5. Evaluate the following binomial coefficients:
   (a) $\binom{5}{2}$
   (b) $\binom{5}{3}$
   (c) $\binom{10}{4}$
   (d) $\binom{11}{7}$

6. In how many ways can you select 5 distinct balls from a box contain 12 balls total?

7. There is a class of 20 students and they need to select a committee of 5 students to plan a party, once these 5 students are picked one needs to be selected to be president of the committee, in how many ways can this be done?

8. There are 12 boys and 18 girls who are eligible to run in a mixed relay. In how many ways could the relay be chosen and they run in a race if the team must contain 2 boys and 2 girls?

9. * Calculate the number of paths from the given pairs of points which which take only take steps right and up *(hint for some of these problems you will have to redraw the grid so that you only move in the right and up directions)*, in the grid below.



(a) What is the number of paths from a to d?

(b) What is the number of paths from a to f?

(c) What is the number of paths from c to e?

(d) What is the number of paths from h to e?

(e) What is the number of paths from f to b?

(f) What is the number of paths from g to h?

(g) What is the number of paths from c to g?

(h) What is the number of paths from a to g?

10. ** Using algebraic manipulations on the definition of the binomial coefficient show that $k \times \binom{n}{k} = n \times \binom{n-1}{k-1}$

11. *** Using algebraic manipulations on the definition of the binomial coefficient show that
$$\binom{n}{k+1} = \binom{n}{k} \times \frac{n-k}{k+1}$$